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## A MATHEMATICAL MODEL FOR THE CALCULATION OF TIME DEPENDENT TERRAIN RADIANCE

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## Abstract

A computerized mathematical model for the calculation of time-dependent terrain surface temperatures and radiances has been formulated and tested. Temperatures are obtained by solving the one-dimensional heat diffusion equation, subject to boundary conditions which account for (1) solar insolation, (2) radiative transfer, (3) convection, (4) rain and water evaporation. Ground vegetation temperatures are calculated by solving a heat balance equation. Required input data include, (1) solar insolation, (2) ambient air temperature, (3) wind velocity, (4) relative humidity, (5) cloud cover, (6) rain intensity. The terrain is configured as successive homogenous layers. The usual form of the output is a computer plot of the time-dependent radiant emittance difference (over a given wavelength interval) between two different terrains subject to the same meteorological conditions. Validity of the model has been established directly by experiment and indirectly through the use of infrared imagery.

## I. Introduction

Despite the high-quality of thermal infrared imagery major interpretation problems still persist. These stem in large measure from the diverse nature and relative importance of the heat-transfer processes which bring about thermal contrast in the natural and man-made objects in the scene and the variety of objects of interest. Indeed most of the interpreter's problems come from the lack of analysis capabilities necessary to assess the relative importance and influence of these processes on a particular object at a particular time. An attempt to provide the interpreter with a computational tool to assist in this aspect has been developed, and is described in the three sections which follow. The discussion is divided into three sections. In Section II a description of the basic model for computing the time-dependent surface temperature is given. Section III deals with the calculation of radiance and scanner response and Section IV provides a brief discussion of the accuracy of the model.

## II. The Temperature Model

To provide valid estimates of the temperature or radiance of a general three-dimensional object in a general environment without introducing very extensive approximations would require much more detailed information about heat-transfer coefficients than is presently available even from laboratory-type measurements. The general one-dimensional case (an open field, a road, building roof, truck top, etc.) is, however, mathematically tractible and can yield valid estimates of the temperatures and radiances. Many objects of interest and practically all backgrounds can be considered planar, hence the importance of the one-dimensional case.

### 1. The Heat Diffusion Equation

The model is based on the assumption that for a large class of objects the most significant heat fluxes are vertical, and that transverse heat flow in the object is negligible. This is surely the case for objects which are uniformly irradiated, whose observable surface normal is essentially vertical, and whose spatial extent is at least comparable to several resolution elements (truck tops, building roofs, road way surfaces etc.). Furthermore, the object's thermal properties are considered to be homogenous, varying only in the vertical dimension (z). Under these assumptions the temperature T(z,t) is given by the solution of the heat diffusion equation,

$$\frac{\partial^2 T(z,t)}{\partial z^2} = \frac{\rho(z)c(z)}{k(z)} \frac{\partial T(z,t)}{\partial t} ,$$

subject to the boundary conditions,

$$\sum_{i=1}^{n} q_{di} = 0 \text{ at } z = 0,$$

and

$$\sum_{i=1}^{m} q_{di} = 0 \text{ at } z = d.$$

The observable surface is $z = 0$, the lower surface is $z = d$, and $\rho(z)$, $c(z)$, and $k(z)$ are the density, specific heat capacity, and conductivity respectively. The $q_i$ represent heat fluxes at the surfaces of the slab object.

The realistic accuracy of this model depends to some extent on the degree to which the thermal characteristics can be approximated by constant values and the physical configuration by successive infinite planar slabs. More importantly, however, it depends on the quantitative accuracy of the approximations for $q_i$ which represent the actual time-dependent heat-transfer processes which take place at the surface exposed to the natural meteorological driving functions.

### 2. Boundary Conditions

Upper Boundary $(z = 0)$. For non-vegetated surfaces there are six essential heat-transfer processes which must be taken into account. These are (a) body conduction, (b) solar absorption, (c) net thermal radiative transfer, (d) convection, (e) rain, and (f) evaporation. The derivation of fluxes appropriate for (c), (d), (e), and (f) have been given elsewhere[1] and here will be described only briefly their individual dependences on the measurable meteorological parameters (ambient air temperature, horizontal wind velocity, relative humidity cloud cover, cloud type, etc.) which are in general time-dependent input data for the model.

a. Body Conduction. This term accounts for the conduction of heat away from the upper surface into the interior of the object. It depends simply on the thermal conductivity of the surface material and the temperature gradient evaluated at the surface;

$$q_{0i} = -k_1 \left.\frac{\partial T(z,t)}{\partial z}\right|_{z=0},$$

where $k_1$ is the thermal conductivity of the surface layer material.

b. Solar absorption. In order to allow for the many different irradiation conditions which are possible in a diurnal cycle this term is an arbitrarily specifiable tabular form which is just the total direct and diffuse solar irradiance multiplied by the total solar absorptivity of the surface multiplied by an obscuration factor. The obscuration factor models the obscuration of the surface (z = 0) from the sun by the overhead foliage canopy and ground cover. This flux is expressed as;

$$q_{02} = -a(1-0_b)S(t)P(0)^{(1+c^2\tan^2\theta)^{1/2}}$$

where
$a$ = effective solar absorptivity,
$0_b$ = fraction per unit area of surface obscured by ground vegetation,
$S(t)$ = total direct and diffuse solar irradiance,
$P(0)$ = probability of a clear line of sight at the zenith through the overhead canopy,
$c$ = canopy characteristic factor,
$\theta$ = angle of the sun from the zenith.

c. Net Thermal Radiation. Net thermal radiation is specified by the difference between the instantaneous total greybody radiation emitted by the surface and that absorbed by the surface from a radiating atmosphere whose mean total emissivity is an analytical function of time and cloud cover and cloud type. The expression is:

$$q_{03} = \varepsilon_0\, \sigma \left[T_0^{\,4} - T_a^{\,4}\left(a+b\sqrt{RH(t)\ P_s(T_a)}\right)\right](1-\nu m),$$

where
$\varepsilon_0$ = thermal emissivity of the surface,
$\sigma$ = Stefan-Boltzmann constant,
$T_0$ = absolute surface temperature, a computed quantity,
$T_a$ = absolute air temperature, a tabular input,
$\nu$ = cloud-type coefficient,
$m(t)$ = cloud cover intensity, a tabular input,
$a$ = atmosphere thermal emissivity coefficient, 0.53,
$b$ = atmosphere thermal emissivity coefficient, 0.47,
$RH(t)$ = fractional relative humidity, a tabular input,
$P_s(T_a)$ = saturated vapor pressure of water in the atmosphere,

d. Convection. The relative contribution of the convection process can vary over several orders of magnitude from a very small rate (molecular conduction) for stable no-wind conditions to a very large rate (forced convection) for unstable atmospheric conditions and large wind velocity. The analytical expression for heat-transfer by convection over terrain which has been used in the model was deduced by using theoretical and empirical formulas and experimental data available in the literature. A complete discussion of the origin of this expression and the determination of the coefficients in it is given in reference 1. The convection term is written in the form of a heat transfer coefficient multiplied by the temperature difference between the air at 160 cm and the surface temperature. The transfer coefficient is a function of the horizontal wind velocity at 160 cm above the surface, a surface roughness parameter, a coefficient which depends on the above mentioned temperature difference, and a so-called atmospheric stability coefficient. Basically this convection term is capable of describing quantitatively the heat-transfer which can take place under the various conditions of stable, neutral, and unstable temperature and wind profile combinations which encompass the conditions from laminar flow to turbulent mixing. For this process the heat flux is of the form:

$$q_{04} = \frac{\gamma\left(\frac{60}{z_0}\right)^{\beta} V(t)\,(T_0-T_a)}{\left(\frac{z}{z_0}\right)\ln\left(\frac{z}{z_0}\right)},$$

where
$T_0$ = surface temperature,
$T_a$ = air temperature,
$z$ = height at which $T_a$ and V measured, usually 160 cm.
$z_0$ = surface roughness parameter,
$\beta$ = stability coefficient

$$= \begin{cases} 1 + .00121(T_0-T_a) & \text{for } T_0 > T_a, \\ 1 + .0115(T_0-T_a) & \text{for } T_0 \leq T_a, \end{cases}$$

$$\gamma = \begin{cases} 0.22 + 5.58\left(\frac{T_0-T_a}{V(t)}\right) & \text{for } T_0 > T_a \\ 0.38 & \text{for } T_0 \leq T_a, \end{cases}$$

$V(t)$ = wind speed, a tabular input.

For "smooth" non-terrain surfaces such as roads, vehicle hoods and tops, roofs etc., a "smooth" convection term is used. This is the standard expression:

$$q_{o4} = h(T_o - T_a),$$

where $h$ = heat transfer coefficient,

$T_a$ = absolute air temperature,

$T_o$ = absolute surface temperature.

e. Rain. The intensity (depth/time) of rain falling on the objects of interest is specifiable as an arbitrary function of time (tabular form). It's immediate thermal effect is accounted for by a temperature equilibriating heat exchange between the water and the surface. The expression used is:

$$q_{o5} = r\rho_w c_w (T_o - T_R),$$

where $r$ = rain intensity, a tabular input,

$\rho_w$ = density of water,

$c_w$ = heat capacity of water,

$T_o$ = surface temperature,

$T_R$ = rain temperature.

f. Rain Evaporation. A certain portion of the rainfall (that remaining after runoff) is susceptible to evaporation. In computing the rate of heat transfer due to evaporation, a form of Dalton's formula is used which accounts for the effects of wind turbulent mixing, and diffusion in the immediate atmosphere. This term depends on the wind speed, the relative humidity, temperature of the air, and the surface (water) temperature. The expression is:

$$q_{o6} = cV(t)^{1/2} \left(P_s(T_o) - RH(t)\, P_s(T_a)\right),$$

where $c$ = constant,

$V(t)$ = wind speed, a tabular input,

$P_s(T_o)$ = saturated vapor pressure of water at the surface temperature,

$P_s(T_a)$ = saturated vapor pressure of water at air temperature,

$RH(t)$ = fractional relative humidity of the air, a tabular input.

Lower Boundary. Two alternate boundary conditions have been programmed on the bottom boundary. One permits arbitrary specification of the temperature of the bottom boundary as a function of time (tabular form). The other allows specification of the total heat flux at the bottom boundary as a function of time (tabular form). The program allows the addition of a constant temperature radiating surface below the bottom boundary for this second case. Total flux specified above is then modified by the radiation between the two surfaces to yield the net flux at the bottom boundary. The expression used is:

$$q_{d1} = q_b(t) + \sigma(k_n \varepsilon_n T_r^4 - k_b \varepsilon_b T_d^4),$$

where $q_b(t)$ = the "rated" total flux output of the heat-source radiating surface, a tabular input,

$\sigma$ = Stefan-Boltzmann constant,

$k_n$ = absorption coefficient and geometric shape factor for the radiating source surface,

$\varepsilon_n$ = thermal emissivity of the radiating source surface,

$T_r$ = absolute temperature of the radiating source surface,

$k_b$ = absorption coefficient and geometric shape factor for the bottom boundary,

$\varepsilon_b$ = thermal emissivity of the bottom boundary,

$T_d$ = absolute temperature of the bottom boundary.

Internal Material Configurations of the Model. The model allows the terrain or object to be described by consecutive layers (maximum of six) of arbitrary thermal properties and arbitrary thickness. Hence, to configure the object or terrain, the thickness, density, specific heat capacity and conductivity of each layer must be specified. In addition, the total solar absortivity, thermal emissivity, and surface roughness of the top layer must be specified.

Initial Conditions. The solution of the partial differential equation requires that the spatial temperature distribution be specified at time $t=t_o$. This distribution is the cumulative result of the effect of the thermal environments which prevailed prior to $t_o$. The response of the object to the environment occuring after $t_o$ will be influenced by the spatial temperature distribution at $t_o$. However, this dependence decreases as time increases and finally becomes negligible after a time comparable to the "time constant" of the system. The model (computer program) uses as input data the time-dependent solar insolation, horizontal wind velocity, ambient air temperature, relative humidity, percent cloud cover, and cloud type, for, say, 5 diurnal cycles. Three or four of these cycles are used to obtain the initial temperature distribution for the start of the fourth or fifth cycle. Thus the response of the object to the input environment on the fourth or fifth cycle accurately reflects the influence of the past thermal history. Experience with the model has shown that three to four cycles is sufficient to establish the thermal history effects on most terrains and high thermal inertia objects.

## 3. The Heat Balance Equation.

The temperature of vegetation is obtained by solving an algebraic heat-balance equation. The leaf of the vegetation is assumed to be a horizontal thin plate with no heat capacity for which the heat fluxes sum to zero i.e.,

$$q_1 - q_2 - q_3 - q_4 = 0,$$

where $q_1 = \sigma a_b \varepsilon_o T_o^4 + a_t[S(t) + \sigma\varepsilon_a(t)]$ ,

$q_2 = \sigma(\varepsilon_t + \varepsilon_b)T_L^4$ ,

$q_3 = C_o(\frac{V(t)}{x})^{1/2}(T_L - T_a)$ or ,

$\frac{c_1}{(x_1)^{1/4}} (T_L - T_a)^{5/4}$ ,

and

$$q_4 = \frac{c_p\rho}{\delta(r_1+r_2)} [P_s(T_L) - RH(t)P_s(T_a)].$$

The symbol definitions are:

$\sigma$ = Stefan-Boltzmann constant,
$a_b$ = absorptivity of the bottom of the leaf,
$\varepsilon_o$ = emissivity of the terrain surface,
$a_t$ = absorptivity of the top of the leaf,
$S(t)$ = total and diffuse solar irradiance, a tabular input,
$\varepsilon_t$ = emissivity of the top of the leaf,
$\varepsilon_a(t)$ = emissivity of the atmosphere,
$\varepsilon_b$ = emissivity of the bottom of the leaf,
$C_o$ = constant shape factor,
$V(t)$ = wind speed, a tabular input,
$x$ = linear dimension of leaf in the wind direction,
$c_1$ = constant shape factor,
$x_1$ = average leaf width over all directions,
$c_p\rho$ = specific heat of air, times density of air,
$\delta$ = composite leaf conversion factor of vapor pressure gradient to concentration gradient,
$r_1$ = leaf boundary layer diffusion resistance,
$r_2$ = leaf bulk diffusion resistance,
$T_o$ = absolute temperature of the surface beneath leaf,
$T_L$ = absolute temperature of the leaf,
$T_a$ = absolute temperature of the air, a tabular input,
$P_s(T)$ = saturated vapor pressure of water at absolute temperature T,
$RH(t)$ = fractional relative humidity, a tabular input.

## III. Radiance Model

### 1. Radiance

Using the computed surface temperatures a separate section of the program is available to convert the temperature data to radiance data and to compute the average radiance of low-lying vegetation and the terrain surface. To calculate the radiance over a given wavelength interval additional information about the surface material must be specified, namely the emissivity as a function of wavelength, and the spectral interval over which the radiance is to be computed. Using this information the following integral is computed:

$$R(T,\lambda_1,\lambda_2) = \int_{\lambda_1}^{\lambda_2} \frac{\varepsilon(\lambda)\ c_1\ d\lambda}{\lambda^5\left[\exp\frac{c_2}{\lambda T} - 1\right]} ,$$

where,

$\varepsilon(\lambda)$ = emissivity, a tabular input,
$c_1$ = constant,
$c_2$ = constant,
$T$ = absolute surface temperature,
$\lambda$ = wavelength.

In the case of leaves, the radiance is computed for the terrain surface and the leaf surface and the two radiances averaged on the basis of the percent of the surface covered by vegetation. The expression is:

$$R_f = O_b R(T_L, \lambda_1, \lambda_2) + (1-O_b)\ R\ (T_o, \lambda_1, \lambda_2),$$

$O_b$ = fraction of surface obscured by leaves,
$T_L$ = temperature of leaf,
$T_o$ = temperature of the terrain surface.

### 2. Scanner Response

When the determination of the relative response of several objects in the scene is to include some estimate of the scanner transfer function, a section of the program can be called upon to compute a scanner response level. The parameters which are common to all optical-mechanical scanning systems and which generally determine the response have been included in this model of the scanner. The main attempt here is to account for the detector wavelength sensivity and filter and atmospheric transmission.

The scanner-detector model can be summarized mathematically by the expression for (as defined here) response:

$$W(t,\lambda_1, \lambda_2)\ C_o \int_{\lambda_2}^{\lambda_1} \frac{D^*(\lambda)\tau(\lambda)\varepsilon(\lambda)c_1 d}{\lambda^5\left[\exp(\frac{c_2}{\lambda T(t)}) - 1\right]},$$

where $C_o = \frac{\omega^2 D^2}{4(A\Delta f)^{1/2}}$

$\omega$ = the instantaneous angular resolution of the system,
$\tau(\lambda)$ = transmission function (object to detector),
$A$ = the sensitive area of the detector,
$D^*(\lambda)$ = spectral D-star of detector,
$D$ = the effective collector diameter,
$\Delta f$ = the electronic bandwidth of the detector and preamp,

$c_1$ = constant,

$c_2$ = constant,

$\lambda$ = radiation wavelength.

## IV. Verification of the Model

Several field experiments have been performed to verify the accuracy of the temperature model. Two of these experiments are described here, and represent the extremes of the results obtained.

From 3 to 7 May 1966, a verification experiment was conducted at Arnold Engineering Development Center (AEDC), Tullahoma, Tennessee, during a period of stable weather and cloudless skies. The site chosen was an untrafficked section of concrete roadway. Meteorological parameter measuring instruments and recording equipment were installed at or near the site to measure solar radiation, air temperature, wind velocity and relative humidity. Engineering drawings of the road and base cross sections were available for the purpose of determining the vertical stratification of materials. No attempt was made to measure the thermal properties of materials except for the solar absorptivity of the concrete surface. Surface temperature measurements were made on 5, 6 May using an Alnor contact pyrometer. The statistical variation of surface temperature was determined by measuring several points across the concrete surface at the same time.

Using the measured values of the meteorological parameters together with reasonable estimates for the unmeasured quantities, a time-varying surface temperature was calculated for the concrete road using the model. A comparison between the calculated temperatures and those actually measured shows the greatest difference between computed and measured temperatures occurs at 0700 hours and is less than 4°F. Such agreement cannot be expected in general.

Another of the field verification experiments was conducted on a concrete aircraft ramp at the Willow Run Laboratories of the University of Michigan from 9-15 July 1966. Some of the meteorological parameters were continuously recorded near the site, but cloud cover, relative humidity, and rainfall were available only as hourly averages. In many cases, instrumentation problems raised doubts about the accuracy of the meteorological data. Unrefined measurements were made of the thermal conductivity and diffusivity of cylindrical cores taken from the concrete, but the surface properties were estimated.

This experiment was especially interesting for two reasons: 1) the weather was extremely variable during the five days, with alternating periods of clear skies and overcast, and several rain showers; and 2) the questionable accuracy of some of the meteorological measurements is indicative of the type of data which would probably be available in an operational use of such a model. The largest discrepancy between the computed temperatures and the measured temperatures for the five days during which the temperatures were measured was at 1200 hours on the last day, at which time the computed temperature exceeded the measured temperature by 13°F. The mean temperature difference over the 23 measured data points was 6.4°F. This difference is within the uncertainty expected of a well-controlled experiment with laboratory-accuracy in the measured meteorological data; and hence well within the expected error from a field-type experiment with its attendant larger measurement errors.

Indirect tests of the validity of the model have been conducted by utilizing extensive thermal infrared imagery available at the University of Michigan. Relative contrast as exhibited in the imagery by various objects and terrains in a given scene has been compared with corresponding computed results from the model. In some cases sequential imagery of the same scene taken every three hours over a complete diurnal cycle was available for comparison with the computed responses. All of these comparisons have indicated no case in which the computed relative ordering of the response from objects and terrains is in disagreement with the observed actual ordering of the corresponding objects in the imagery.

## V. Conclusion

Sufficient evidence has been obtained to establish that man-made and natural objects usually observed in infrared thermal imagery can be modeled, and that the model can be relatively simple with respect to the diverse nature of potentially important heat-transfer processes occuring on or near the terrestrial surface. It is further evident that the model is sufficiently realistic to be of significant utility to present and future users of infrared scanners, especially in connection with applications in scientific exploration and remote sensing.

## References


1. Bornemeier, D. and Horvath, R., Infrared and Photo Record Analysis, Final Report, Vol. I: A Mathematical Predictive Model for Target Temperature as a Function of Environment, (Infrared and Optical Sensor Laboratory, Willow Run Laboratories, The University of Michigan, Ann Arbor, Michigan, November 1966), Rep. 7417-14-F.

2. Bornemeier, D., Temperature Predictive Model Development, Final Report, (Infrared and Optical Sensor Laboratory, Willow Run Laboratories, University of Michigan, Ann Arbor, Michigan, to be published June 1969.)